THE PUMPING CHARACTERISTICS OF SCREW ROTORS III. VERIFICATION OF EQUATION FOR THE CALCULATION OF THE PUMPING CAPACITY OF SCREW ROTORS

František RIEGER

Faculty of Mechanical Engineering,
Czech Technical University, 166 07 Prague 6

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The paper deals with the verification of certain simplifying assumptions introduced in the derivation of the equations for the pumping characteristics of screw rotors[1]. For this purpose data have been used, on the one hand, taken over from the literature, on the other hand, those obtained in this communication[2]. A method has been recommended for the calculation of the pumping characteristics of screw rotors rotating in a barrel.

Numerous measurements have been published in the past already carried out with the aim to verify the theoretical calculations of the pumping capacity of screw rotors. The first experiments, carried out on screws with relatively small depth of the screw channel, H/W, by Rowell and Finlayson[3], in general confirmed their planar theory for the calculation of the pumping capacity. This theory was also satisfactorily confirmed by measurements of maximum pressure, carried out on deeper screws by Pigot[4], namely because there is no appreciable effect of the screw blade on its value (the coefficients F_d and F_p, which have approximately the same value, factor out). The measurements of the pumping capacity carried out by McKelvey[5], however, confirmed that an account for the effect of the side walls improves the agreement between the experiment and the theory. The side walls of the channel affect significantly the pumping capacity, particularly at free discharge, as shown by measurements of Squires[6], carried out with the aim to confirm Squires own method of correction. The presented results of a relatively small number of measurements, however, do not permit a more reliable verification of all assumptions introduced in the derivation of the theoretical equation for the pumping characteristic, which is the main goal of this paper.

The Verification of the Effect of the Side Walls in Channels with Non-negligible Curvature

The correction coefficients on the effect of the side walls, F_d and F_p, were derived from the case of the flow in a straight channel of rectangular cross section. In the calculation of the pumping characteristic of a screw rotor it is, however, assumed that the effect is respected of the side walls even in the case of the screw channel. For the verification of the accuracy of this assumption it is possible to use two earlier published papers, namely works devoted to the flow between discs rotating in a stationary cylinder[7], or the results published in Říha's thesis[8].

The shaft with the discs rotating in a stationary cylinder is depicted in Fig. 1. This configuration actually represents the extreme case of a screw with zero pitch. In paper[7], devoted to the flow in this system, relationships are presented for the calculation of the flow rate under the drag as well as pressure flow in this configuration. As long as the flow rate in the annulus between the discs is expressed from the equation for two unconfinned plates, derived in ref.[1], the following relationship is obtained

$$\dot{V} = \frac{UHW}{2}\bar{F}_d - \frac{\Delta p W H^3}{12\mu L_z}\bar{F}_p . \qquad (1)$$

This equation represents a definition expression for the correction coefficients $\bar{F}_d$ and $\bar{F}_p$.

The flow rate between the discs, however, could have been determined also from Eq. (31) of ref.[1] for the calculation of screw rotors which in this case takes the following form

$$\dot{V} = \frac{UHW}{2}F_d F_{dc} - \frac{\Delta p W H^3}{12\mu L_z}F_p F_{pc} . \qquad (2)$$

The values of the correction coefficients F_{dc} and F_{pc} for zero pitch may be determined from Fig. 7 or Eqs (24) and (25) presented in the first part of this communication[1].

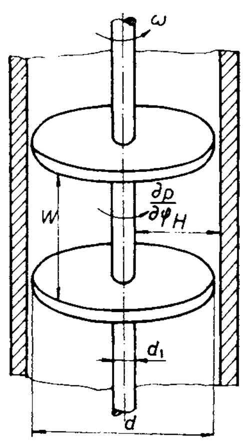

FIG. 1
Experimental set-up

A comparison of the values $\bar{F}_d$ and $\bar{F}_p$, calculated from the relations for the flow rate between the discs[7], with the values of the coefficients F_dF_{dc} or F_pF_{pc}, calculated from the expression presented in ref.[1], is furnished in Table I. From the ratio of both compared values shown in the table it follows that the agreement of the calculation according to Eq. (*2*) with the reality represented by Eq. (*1*) is good with the exception

TABLE I

A comparison of the coefficients $\bar{F}_d$ and $\bar{F}_p$ with the values of the products F_dF_{dc} and F_pF_{pc}

d_1/d	H/W	$\bar{F}_d$	F_dF_{dc}	$F_dF_{dc}/\bar{F}_d$	$\bar{F}_p$	F_pF_{pc}	$F_pF_{pc}/\bar{F}_p$
0·1	0·1	1·005	1·002	0·997	1·509	1·496	0·991
0·1	0·25	0·924	0·916	0·991	1·378	1·345	0·976
0·1	0·5	0·789	0·773	0·980	1·161	1·095	0·943
0·1	0·75	0·660	0·640	0·970	0·954	0·864	0·906
0·1	1·0	0·548	0·530	0·967	0·773	0·673	0·871
0·1	2·5	0·230	0·230	1·00	0·252	0·191	0·758
0·1	5·0	0·112	0·115	1·027	0·0797	0·0558	0·700
0·1	7·5	0·0737	0·0767	1·041	0·0384	0·0260	0·677
0·1	10·0	0·0550	0·0575	1·045	0·0225	0·0150	0·667
0·5	0·1	1·017	1·018	1·001	1·232	1·230	0·998
0·5	0·25	0·928	0·930	1·002	1·110	1·106	0·996
0·5	0·5	0·782	0·785	1·004	0·908	0·901	0·992
0·5	0·75	0·646	0·650	1·006	0·721	0·711	0·986
0·5	1·0	0·531	0·538	1·013	0·565	0·554	0·981
0·5	2·5	0·224	0·233	1·040	0·161	0·157	0·963
0·5	5·0	0·110	0·117	1·064	0·0480	0·0459	0·956
0·5	7·5	0·0731	0·0778	1·064	0·0224	0·0214	0·955
0·5	10·0	0·0547	0·0584	1·068	0·0123	0·0123	0·953

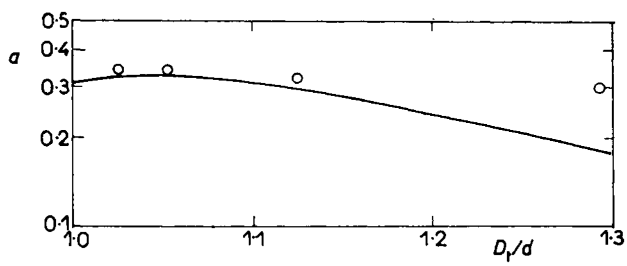

FIG. 2

The effect of the clearance on the drag flow, solid line indicates the computed dependence (Eq. (*33*) of ref.[1])

of the pressure flow in deep channels ($H/W > 0{\cdot}5$) with large curvature ($d_1/D_t = 0{\cdot}1$). The pressure flow in such configurations is in fact much more intensive which could lead, calculating according to Eq. (*2*), to insufficient dimensioning of screw rotors with relatively small diameter of the root and the pitch. For such geometries it is more proper to use for correction of the effect of side walls the coefficients F_d and F_p calculated[7] from the case of the flow between the discs.

An alternative way of verifying the equation for the calculation of the dimensionless pumping capacity

$$\frac{\dot{V}}{nd^3} = a - b\,\frac{\Delta p}{\mu n} \qquad (3)$$

is a comparison of the values of the constants calculated on the basis of Eq. (*3*) presented in the first part of this communication[1] with the values a and b computed numerically for selected geometries under the stabilized flow at zero clearance by Říha[8]. This comparison is furnished in Table II. The table shows the good agreement of the values calculated according to the proposed equation with the values obtained by numerical solution. The only exception to this is the last geometry with extremely high pitch which, however, practically does not exist in industrial applications.

From both above presented comparisons carried out for zero clearance and stabilized flow it follows that the method of respecting the effect of the side walls and curvature[1] by its accuracy satisfies the requirements put on the technical calculations of screw rotors. For this reason the attention in the subsequent part of this paper has been devoted particularly to experimental verification of the remaining effects, namely the effect of end regions of flow development and the effect of the clearance between the rotor and the barrel.

TABLE II

A comparison of the values computed from Eq. (*33*) of ref.[1] with the results of the numerical calculation[8]

Geometrical parameters			Numerical calculation[8]		Calculation from Eq. (*33*) of ref.[1]	
d_1/d	s/d	e/d	a	$b \cdot 10^4$	a	$b \cdot 10^4$
0·231	0·55	0·0577	0·177	0·321	0·173	0·307
0·231	1·15	0·0577	0·428	1·68	0·423	1·70
0·231	1·69	0·0577	0·540	3·07	0·524	3·13
0·231	3·17	0·0577	0·544	5·56	0·516	5·84
0·231	5·41	0·0577	0·473	6·98	0·383	7·39

Experimental Verification of the Effect of Flow Development

For the verification of the effect of the development of the flow we used results of measurements on the stationary screw presented in the second part of this communication[2]. In case of negligible effect of flow development in the end regions of the rotors one could calculate the overall pressure difference across the screw as a product of the pressure gradient in region of stabilized flow and the length of the screw

$$\Delta p = (\partial p/\partial l)_s L \tag{4}$$

or, in the dimensionless form, as

$$\frac{\Delta p d^3}{\mu \dot{V}} = \left[\left(\frac{\partial p}{\partial l}\right)_s \frac{d^4}{\mu \dot{V}}\right] \frac{L}{d}, \tag{5}$$

where the expression in the brackets represents the dimensionless pressure gradient. Its values measured for individual configuration were tabulated in Table IV of the second part of this communication[2].

A comparison of the measured values of the dimensionless pressure difference with the values computed from the experimental values of the dimensionless pressure gradient according to Eq. (5) is furnished in Table III. The ratio of both values in the last but one column of the table thus expresses the effect of the end regions on the pressure flow equally as the value of the correction coefficient F_{pe}. The values of the correction coefficient computed from Eq. (29) of ref.[1] are given in the last column of the Table III. While the values of the coefficient F_{pe} are always higher than unity, the values of the ratio, given in the last column of Table III, fluctuated unsystematically around unity but mostly do not differ significantly from unity. From this it can be inferred that the effect of end regions may be, at least in case of the investigated screws, neglected. This finding may be explained by the development of the pressure profile, which, according to the analysis of Booy[9], increases for shallow screws the flow rate from the pressure flow, but, for deeper channel screws, is compensated by the development of the velocity profile, which, on the contrary, decreases the flow rate.

Experimental Verification of the Effect of the Clearance between the Screw and the Barrel

The effect of the clearance on the convective flow may be judged from comparison of the values of the constant a in Eq. (3) measured on individual screws. This comparison is shown in Fig. 2 for screw number 3 showing the dependence of the values of this constant on the ratio of the diameter of the barrel to the diameter of the screw, D_t/d. The figure shows also by a solid line the curve computed for rotor 3 on the basis of Eq. (*33*) given in ref.[1] (for $F_{de} = 1$). From the figure it is apparent that at the ratio $D_t/d < 1{\cdot}13$ the theoretical equation fits the measured data fairly well. Only the value measured on the barrel D, with maximum clearance, is significantly higher than the computed value. Clearances of this magnitude, however, are not encountered in practice, and if such situation occurs, one can utilize the fact that the dependence of a on D_t/d is relatively weak and the constant a may be computed from the theoretical equation for the case without clearance ($D_t = d$). The same conclusion may be arrived at also on the basis of data measured for the other rotors.

The effect of the clearance on the pressure flow may be ascertained from the values measured on both experimental set-ups. In doing so we shall start from the equation of the pumping characteristic in the form (see Eq. (*35*) in ref.[1])

$$\frac{\Delta p}{\mu n} = A - B \frac{\dot{V}}{nd^3}, \qquad (6)$$

where the effect of the pressure flow is characterized by the constant B. On multi-

TABLE III

A comparison of the experimental and calculated values of F_{pe}

Screw	Barrel	$f_1 = \frac{\Delta p d^3}{\mu \dot{V}}$	$f_2 = \left(\frac{\partial p}{\partial l}\right)_s \frac{d^4}{\mu \dot{V}} \frac{L}{d}$	f_2/f_1	F_{pe}
1	a	49 530	49 866	1·007	1·07
1	b	30 067	29 582	0·984	1·08
1	c	13 031	12 395	0·951	1·08
2	a	21 490	22 444	1·044	1·07
2	b	17 454	17 211	0·986	1·07
2	c	8 190	9 383	1·146	1·07
3	a	11 533	13 006	1·128	1·12
3	b	8 754	9 272	1·059	1·12
3	c	5 373	5 928	1·103	1·12
4	a	4 168	4 167	1·000	1·10
4	b	3 219	3 528	1·096	1·10
4	c	3 085	3 000	0·972	1·10
5	a	6 451	6 911	1·071	1·11
5	b	4 816	5 000	1·038	1·12
5	c	3 374	3 094	0·917	1·12
6	a	2 402	2 433	1·013	1·08
6	b	2 116	2 094	0·990	1·08

plying Eq. (6) by the frequency of revolution we shall obtain

$$\frac{\Delta p}{\mu} = An - B\frac{\dot{V}}{d^3}\,. \tag{7}$$

For the stationary screw the frequency of revolution is zero and upon considering the opposite direction of pressure difference Eq. (7) may be written in the following form

$$B = \frac{\Delta p d^3}{\mu \dot{V}}\,. \tag{8}$$

The value of the costant B is thus equal the pressure difference measured with the stationary screw and may be also computed from the dimensionless pressure gradient measured on the stationary screw according to Eq. (5).

On the basis of the values of the constant B measured with various barrels on the rotating as well as the stationary screw one can thus judge the effect of the clearance on the pressure flow. The obtained dependence of B on the ratio D_t/d for the rotor **3** is shown in Fig. 3. The figure also shows by a solid line the dependence computed from Eqs (33) and (36b) from ref.[1] (for $F_{pe} = 1$). From the figure it is apparent that the computed curve fits the experimental values with acceptable accuracy only for $D_t/d < 1{\cdot}06$. Its use for greater clearances would lead to seriously conservative dimensioning of screw rotors.

From Fig. 3 it is, however, also apparent that the dependence of the parameter B on the ratio D_t/d may be approximated in the whole range of D_t/d values in semi-log coordinates by a straight line passing through the point $D_t/d = 1$, $B = B_0$, where B_0 designates the value of B computed for screws with no clearance. This is true also for the other screws and, accordingly, the slopes of the straight line dependences

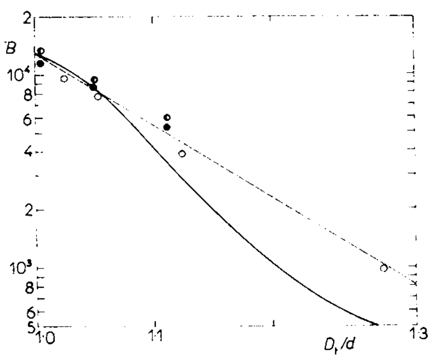

FIG. 3

The dependence of the dimensionless pressure difference on the ratio D_t/d. Solid line indicates the computed dependence (Eq. (33) of ref.[1])

of B on D_t/D in semi-log coordinates passing through the point $D_t/d = 1$, $B = B_0$ were evaluated by linear regression from experimental data for individual rotors. These values were then analyzed in the log-log coordinates by multiple regression in dependence on d_1/d and s/d. This way following empirical correlations were obtained describing for the investigated rotors the effect of the clearance on the pressure flow

$$B = B_0 \exp\left[16{\cdot}26(1 - D_t/d)(s/d)^{-0.48}(d_1/d)^{0.82}\right]. \tag{9}$$

For the calculation of the pumping characteristics of screw rotors one can thus recommend, based on the analysis of the above experimental results, the following semiempirical equation

$$\frac{\dot{V}}{nd^3} = a_0 - b_0 \exp\left[16{\cdot}26(D_t/d - 1)(s/d)^{-0.48}(d_1/d)^{0.82}\right]\frac{\Delta p}{\mu n} \tag{10}$$

valid in the range of geometrical parameters of the investigated rotors. The values of the coefficients a_0 and b_0 for the case with no clearance ($D_t = d$, $k_d = k_p = 0$) may be computed from the following theoretical equation

$$\frac{\dot{V}}{nd^3} = \frac{\pi}{4}\left(\frac{s}{d} - i\frac{e}{d}\right)\left(\frac{D_t}{d} - \frac{d_1}{d}\right)\frac{D_t}{d}\cos^2\varphi_t F_d F_{dc}(1 - k_d) -$$

$$- \frac{1}{96}\left(\frac{s}{d} - i\frac{e}{d}\right)\left(\frac{D}{d} - \frac{d_1}{d}\right)^3 \frac{d}{L}\sin\varphi_t \cos\varphi_t F_p F_{pc}(1 + k_p)\frac{\Delta p}{\mu n} \tag{11}$$

which may be recommended only for the calculations of screws characterized by the ratio satisfying the constraint $D_t/d < 1{\cdot}06$. For higher values of the D_t/d ratio one has to use Eq. (*10*) as Eq. (*11*) may lead, particularly in cases with highly intensive pressure flow, to overdimensioning of the designed screws.

LIST OF SYMBOLS

a	coefficient in Eq. (*3*)
b	coefficient in Eq. (*3*)
A	coefficient in Eq. (*6*)
B	coefficient in Eq. (*6*)
d	diameter of screw rotor, m
d_1	diameter of screw root, m
D_t	internal diameter of barrel, m
e	axial thickness of screw blade, m
F	correction coefficient
H	depth of screw channel, m
i	number of threads on the screw

k	correction coefficient on clearance
l	axial coordinate, m
L	length of screw rotor, m
L_z	length of screw channel, m
n	frequency of revolution, s^{-1}
p	pressure, Pa
s	pitch of screw, m
U	component of peripheral velocity in direction of channel, m s^{-1}
$\dot{V}$	volumetric flow rate, $m^3 s^{-1}$
W	width of screw channel, m
φ	helix angle
μ	dynamic viscosity, Pa.s

Subscripts

d	drag flow
p	pressure flow
c	curvature
e	end effects
s	stabilized
t	on diameter D_t

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